

Fostering Authentic, Sustained and Progressive Mathematical Knowledge-Building Activity in CSCL Communities

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In this paper, we report on a study where students engaging in model-eliciting mathematical problems with collective discourse mediated by Computer Supported Collaborative Learning (CSCL) software achieved the kind of authentic, sustained, and progressive knowledge-building activity that until now had not been achieved in CSCL software-mediated mathematics communities.

Background to the Research

A major pursuit of most mathematicians working in the living discipline of mathematics is the production and improvement of mathematical conceptual artefacts such as mathematical models. In order to do this, mathematicians often form research communities who invest their resources in the collective pursuit of understanding by engaging in:

- knowledge-building discourse where problems are formulated and posed, assumptions are identified and evaluated, conjectures are made, and findings are shared and critiqued; and
- the investigation of authentic mathematical problems which advances the corpus of knowledge within the discipline of mathematics by: producing new conceptual artefacts and/or by improving existing conceptual artefacts.

This contrasts with the sort of mathematical activity most children engage in within their classrooms. Most classroom mathematical activity tends to be a severely atrophied version of the mathematical activity engaged in by mathematicians (Bereiter, in press; Lesh & Doerr, in press).

According to Bereiter (in press), the major focus of students in most mathematics classrooms is the completion of tasks (e.g., the completion of worksheet and textbook exercises). He, therefore, advocates that mathematics classrooms be reformulated as knowledge-building communities where students are engaged in authentic mathematical activity and produce mathematical conceptual artefacts that can be discussed, tested, compared, and hypothetically modified. In these communities, students see their main job as producing and improving such artefacts, not simply the completion of school tasks.

Information technology has been recognised as an important tool for supporting knowledge-building communities in classrooms (Scardamalia & Bereiter, 1996). However, because most existing information technology-based educational materials (e.g., CAI, ITS, educational games, simulations etc.) tend to foster neither collaborative learning nor knowledge-building (Bereiter, in press), Computer Supported Collaborative Learning (CSCL) software systems such as *Knowledge Forum*® (Scardamalia & Bereiter, 1998) have been designed. *Knowledge Forum*® is a communal multimedia database into which students may enter various kinds of text or graphic notes. When off-line, students plan

knowledge-building projects, seek information from a variety of sources, and engage in small group and whole class discussions of questions, ideas and findings. When on-line, students describe plans, enter new information through text and graphic notes, and participate in more pointed discourse on questions, ideas and findings. Constructive commenting on other students' notes is proactively encouraged and has been shown to be effective in facilitating the construction of conceptual artefacts (Woodruff & Brett, 1993).

However, establishing and maintaining knowledge-building communities of practice with *Knowledge Forum*® (and its earlier version called *CSILE*) in the domain of mathematics has been found to be a rather intractable problem (De Corte, Verschaffel, Lowyck, Dhert, & Vanderput, 1999; Nason, Brett & Woodruff, 1996; Scardamalia & Bereiter, 1996). This contrasts with the experiences in other discipline areas such as science (Brett & Woodruff, 1993; Scardamalia & Bereiter, 1996).

One possible reason for why researchers have had more success in establishing and maintaining CSCL-mediated knowledge-building communities in science classrooms could be due to qualitative differences in the nature of the problems being investigated in the mathematics and science classrooms.

Credence for this viewpoint is provided by recent research conducted by Lesh and his colleagues into the design of "Real Life" Model-Eliciting Activities (Lesh, Hoover, Hole, Kelly, & Post, 1999). In Model-Eliciting activities, the products that students produce include more than answers to questions; they involve producing models or other conceptual artefacts for constructing, describing, explaining, manipulating, predicting and controlling complex systems. Based on their analysis of activities which were most effective in eliciting high levels of symbolizing, communicating and mathematizing, Lesh et al. (1999) developed the following six principles to inform the design of good model-eliciting and thought-revealing activities:

1. *The Personal Meaningfulness Principle* (sometimes called the reality principle)
2. *The Model Construction Principle*: Does the task create the need for a model to be constructed, or modified, extended or refined? Is attention focused on underlying patterns and regularities?
3. *The Self-Evaluation Principle*: Will students be able to judge for themselves when their responses are good enough? For what purposes are the results needed? by whom? when?
4. *The Model-Documentation Principle*: Will the response require students to explicitly reveal how they are thinking about situation? What kind of system (mathematics objects, patterns, regularities) are they thinking about?
5. *The Simple Prototype Principle*: Will the solution provide a useful prototype (or metaphor) for interpreting a variety of other structurally similar situations?
6. *The Model Generalisation Principle*: Can the model generated be applied to a broader range of situations? Is the model reusable, sharable and modifiable?

A close analysis of the science problems used in the studies where CSCL software systems have been very successful in establishing and maintaining science knowledge-building communities (e.g., Woodruff & Meyer, 1995; Woodruff and Meyer, 1997) reveals that the science problems more than adequately meet Lesh et al.'s six principles. These problems were effective in eliciting high levels of authentic science activity such as hypothesising, symbolizing, communicating and "scientifizing". In contrast however, most of the problems used to promote software-supported mathematical knowledge-building communities (e.g., De Corte et al., 1999; Nason et al., 1996) do not adequately meet this

set of six principles. Thus, in hindsight, this may explain why the studies were unlikely to be effective in eliciting high levels of symbolizing, communicating and mathematizing.

Therefore, in the study being reported in this paper, the major aim was to investigate whether having students engage in model-eliciting mathematical problems with collective discourse mediated by Knowledge Forum would achieve the kind of authentic, sustained, and progressive knowledge-building activity that has been achieved in more content-rich discipline areas such as science.

Method

Participants

The twenty-one participants in this study were students from a Grade 6 class at a private urban Canadian school for girls. The twenty-one students in the class were assigned to seven mixed-ability groups of three students.

Data Sources

The investigation involved detailed analysis from four data sources: (1) Observation notes from the lessons; (2) Unstructured on-the-run interviews in which students were asked to explain their models, how they had created their models, and what they had learned, (3) Students' spreadsheet models; and (4) Knowledge Forum notes.

Procedure

Initially, the teacher/researcher presented the students with an article "Best City in the North-East: Providence RI" (Money, 1998) that explained why Providence had won "Money" magazine's award for being the best city in the north-east of the United States. A whole class discussion based around a set of four focus questions followed the reading of the article. The purpose of these questions was to help the students "read with a mathematical eye" while also familiarizing them with the context of the model-eliciting activity – so that their solutions would be based on extensions of the students "real life" knowledge and experiences (Lesh, Cramer, Doerr, Post, & Zawoiewski, in press). After the discussion based around the set of four questions, the students were presented with the following problem:

Ms Estrina has recently emigrated from Russia. Although she has been living in Toronto for about a year, she has not yet made up her mind about where she would like to live in Canada. In order to help her decide, she wants to create a table ranking Canadian cities in terms of quality of life. Your task is to create a model that will enable her to create this table.

The production and the revision of the systems proceeded in four phases.

Phase 1: Initial model-eliciting activity. After the problem had been introduced, the whole class engaged in a "brainstorming" session in which they identified and briefly discussed some of the criteria that would need to be considered when one was ranking cities (e.g., safety, health services, educational services).

Following the brainstorming session, the class broke up into their seven groups. Each group then set about producing their initial model. They did this by first listing all the criteria that could be used for ranking the cities. Because their lists contained more than ten

criteria, each group of students then decided to organise their list of criteria into a set of five or six categories. For example, Kath's group categorised criteria such as multiculturalism, variety of ethnic foods, cultural celebrations under the rubric of "culture". Anne's group categorised criteria such as safety, education, multiculturalism and leisure under the rubric of "quality of life".

Once they had decided on their set of categories, each group of students then entered their set of categories into the top row of an Excel spreadsheet. After they had entered their categories into the spreadsheet, each group of students listed 6-8 Canadian cities in the left hand column of the spreadsheet (see Figure 1 below).

	Culture	Law Enforcement	Entertainment	Government	Attractions	Transportation
Montreal						
Toronto						
Victoria						
Ottawa						
Charlottetown						
Calgary						
Yellowknife						

Figure 1. Initial layout of spreadsheet models.

After they had entered the categories and cities into the spreadsheet, each group then used books from the library and internet databases to gather information with respect to each of categories for each of the cities. Based on this information, they then awarded each city a score out of ten for each category. Following this, they used the spreadsheet to generate a total score for each city and then to rank the cities in descending order according to their total scores.

Phase 2: Sharing of initial models. During this phase, each group of students presented their initial model to other members of the class. The other members of the class were invited to make comments and/or ask questions about the model.

Phase 3: Revision of models. Following the sharing of the initial models, each group then proceeded to further explore and modify its spreadsheet model. During the sharing of the initial models, there was much debate about whether each category should be given a score out of ten. As a consequence, all of the groups then decided that the categories needed to be prioritised. Some groups did this by allocating more points for some categories than others (e.g., scoring culture out of five and transportation out of ten). Other groups did this by building a weighting into the formula used to generate the total score (e.g., multiplying the score out of ten for culture by five and the score out of ten for transportation by eight).

As this was occurring, it was noted that each group of students made sure they were aware of what ideas the other groups were investigating by intermittently "scouting" (Meyer & Woodruff, 1997) around the room looking at and talking to other groups about their computer spreadsheet models. This informal, "scouting" form of discourse played a very important role in facilitating the progress in each group's model. For example, some groups changed their weightings after their scouting forays.

Phase 4: Dissemination and revision of models in Knowledge Forum. This phase began with each group posting their revised model on the *Knowledge Forum*® shared database.

Then, during the next three sessions, each group of students read and provided constructive feedback (such as comments, questions, propositions) to other groups about their models. Based on the feedback from other groups and the teacher/researcher, each group also iteratively revised their own model. In order to facilitate the process of constructive feedback, each session began with a brief whole-class discussion about how to provide constructive feedback. The classroom teachers further reinforced this instruction during other classroom activities.

Results and Discussion

Three important elements of knowledge-building activity that have been noted in mathematical research communities were observed during the course of the study: (1) the posing and exploration of conjectures, (2) the collective pursuit of understanding of key mathematical concepts, and (3) the incremental improvement of the mathematical models.

Table 1

Knowledge-Building Activity of Students

Elements of knowledge-building	Exemplars
Conjecturing	• Changes to weightings and definitions
Collective pursuit of understanding	• Investigation of meanings of numbers
Incremental improvements of models	• Weighting of scores • Explanation of how and why scores were generated

Conjecturing

During mathematical investigations, mathematicians often engage in the process of conjecturing. For example, they often investigate “what-if” some of the assumptions underlying the solutions to the problems they are investigating are changed (Association of Symbolic Logic Association, 1998). This often can lead to the alternative means for solving the problems being identified and to the development of better mathematical models.

When the students generated their initial models in Phase 1, they awarded a score out of ten for each of the categories. They assumed that this was the “only correct way” by which to generate valid scores for each city. During the sharing of the initial models in Phase 2, a few of the students challenged this assumption and argued that because some categories were more important than other categories, different weightings should be apportioned to each of the categories. The whole of the class accepted this argument. Therefore, during Phase 3, each group of students experimented with different weightings for each of the categories and investigated how this influenced the rankings of the cities.

As they engaged in this “what-iffing” activity, the students often opened up new vistas for investigating the problem. For example, some of the groups relooked at their definitions of each of the categories and generated new definitions. This not only enabled them to better justify the sets of weightings they wished to adopt for their models, but also it gave them the option of modifying their models by revising definitions of categories rather than by revision of weightings. Whilst engaged in this type of conjecturing activity, the students gained deeper insights into the nature of ranking tables; many came to realise the subjective nature of ranking tables which in turn led them to question the validity of some ranking tables they had found in the mass media.

Collective Pursuit of Understanding

As the students engaged in the iterative revisions of their models, it was noted that each of the groups continuously delved deeper into the meaning of the scores their models generated for each category. This was reflected in the qualitative changes made to the explanations of how the models generated the scores for each of the categories.

During the earlier iterations of their models, each group of students usually just indicated the total number of points to be allocated for each category and what factors they had considered when generating the scores for each city. For example, Ayn's group initially provided the following explanation of their model's scoring schema for "Quality of Life":

Quality of living is how good life is in the city: safety, education, multiculturalism, leisure (recreation) etc. Score out of 10.

Like all of the other initial explanations provided by the groups, this explanation did not provide details about: 1) how the score for each city had been generated, or 2) what a score such as $\frac{6}{10}$ meant (e.g., is it a poor, average or quite a good score).

As they continuously received comments, propositions and questions from other groups of students and their teachers during Phases 3-4, the explanations were elaborated on and when necessary clarified so that the readers were provided with more details about how the models generated the scores and what the scores meant. For example, by its final iteration, Ayn's group had modified the explanation of their model's scoring schema for "Quality of Life" to:

Quality of living is how good life is in the city: safety, education, multiculturalism, leisure (recreation) etc.

How to Determine the Score: If the city was in the top six of the twenty-four cities for its crime rates, then we gave it ten points. If the city was between seven and twelve for crime rates, we gave it seven and a half points. If the city was between thirteen and eighteen for crime rates we gave it a score of five. If it was between nineteen and twenty-four we awarded it two and a half points.

We added on three points for good entertainment; two points for fairly good entertainment, and one point for poor entertainment. Next, we gave the city a score out of three for education based on the population. We divided the cities into categories based on the population: millions, hundred thousands, and ten thousands. Points were awarded, giving the cities with the highest number of number of secondary school graduates highest number of points in each of the categories. Three points were added for good multiculturalism, two for fairly good, and one for poor.

The score is out of nineteen. /19

Another qualitative change that occurred to the explanations was that many of the groups utilised If-Then conditional statements in the later versions of their models to indicate that a score generated by their model for any category would be based not only on objective information (e.g., average temperatures etc.) but also on the personal preferences of the person using their system. A good example of this is provided by Mary's group when they explain how Ms Estrin could use their model to generate scores for "Environment:

Environment means the climate, biome, and landforms. The climate should meet what Ms. Estrin likes. If she likes warm climates, maybe she could move to the prairies; if she likes cold climates, maybe she will like Iqaluit. The biome will depend on which part of the country you are. If Ms. Estrin likes a temperate biome, Toronto would be a nice place...

Incremental Improvement of Models

During the course of the study, the models developed by the groups of students were incrementally improved and became mathematically more complex and sophisticated. This was exemplified in the final versions of the models by the:

1. different methods utilised in the models to weight the scores for each category
2. complex schemes developed for scoring each of the categories
3. detailed explanations of the meaning of a score
4. inclusion of If-Then conditional statements in the explanations that enabled users of many of the models to utilise both objective and subjective factors when generating a city's score for a category.

This realisation by most of the groups that subjective factors play very important roles in ranking table models such as "Which is the Best City in Canada" was probably one of the major factors that led to the improvements made to the models.

Conclusions and Future Directions

The students in this study engaged in authentic, sustained, and progressive knowledge-building activity very similar to that which has been achieved in more content-rich discipline areas such as science. Indeed, the quality of their symbolizing, communicating, mathematizing and collective pursuit of understanding had much in common with that observed in many mathematical research communities.

Much of the success we had in establishing and maintaining the mathematics knowledge building community in this study can be attributed to the rich context for mathematizing provided by the problem. However, when analysing why this might be, we realized we needed to modify Lesh et al.'s (1999) six principles. Specifically, both Principle 1 (*personal meaningfulness principle*) and Principle 3 (*the self-evaluation principle*) need to be modified to address the collective goals of the community. Therefore we postulate that the personal aspects of principles 1 and 3 be modified as follows:

Collectively meaningful principle: Will members of the community be able to collectively make sense of the situation?

Negotiated collective-evaluation principle: Will all members of the community be able to judge for themselves when the responses are good enough? For what purposes are the results needed? by whom? when?

Further, the Knowledge Building community provided contexts and scaffolds for both intra- and inter-group discourse during the construction and iterative revisions of the models. It enabled the best ideas to surface, be explored and evaluated and then modified/improved. It also enabled the students to identify limitations in their models. For example, Kate and her group presented a model in which safety and environment received the highest weightings. They felt that they had created a final version of their model until it was pointed out to them that their model would rate cities like Flin Flon, in northern Manitoba (a mining community of approximately 7000 people at 54 degrees north latitude) higher than Toronto and Vancouver. Flin Flon has little crime and automobile pollution; however, it also has no movie theatre complex, no public transportation and no shopping malls! After receiving this feedback, Kate and her group rapidly and extensively modified their model to take cognisance of factors such as these. The community also provided the opportunity to see other groups demonstrating varying degrees of success. This peer-

modelling promoted the thinking of all, provided examples of success and moved the class ahead--often overcoming impassés that can stall the efforts of groups working alone.

In the near future, students in an Australian school will be posed a similar problem where they will be required to generate a model for ranking Australian cities. Then via the means of *Knowledge Forum*®, the Canadian and Australian students will comment on one another's models and iteratively revise their models based on these comments. This will provide us with opportunities to engage in further analysis of inter-group data--with a condition that looks at cross-country/cultural configurations as a means of assisting us in refining the pedagogical approach (Woodruff, 2001) and also with opportunities to evaluate and verify/modify the altered set of principles for the design of model-eliciting mathematics problems that can provide the context for establishing and maintaining mathematics knowledge-building communities.

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